Math 115

Spring 2017

Lecture 13

Quick Review

① Write $4x - 3y = 9$ in Slope-Int. form

$-3y = -4x + 9$

$y = \frac{-4}{-3}x + \frac{9}{-3}$

$\Rightarrow \boxed{y = \frac{4}{3}x - 3}$

② Find d, M, and m for $\overline{AB}$ where

$A(-6, 0)$ and $B(0, 8)$.

$d = \sqrt{(-6-0)^2 + (0-8)^2}$

$= \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = \boxed{10}$

$M\left(\frac{-6+0}{2}, \frac{0+8}{2}\right)$

$= \boxed{M(-3, 4)}$

$m = \frac{0-8}{-6-0} = \frac{-8}{-6} = \frac{8}{6} = \boxed{\frac{4}{3}}$

find eqn of a line **parallel** to $y = \frac{3}{4}x - 2$
that contains the point $(4, 0)$.

$$y - y_1 = m(x - x_1)$$

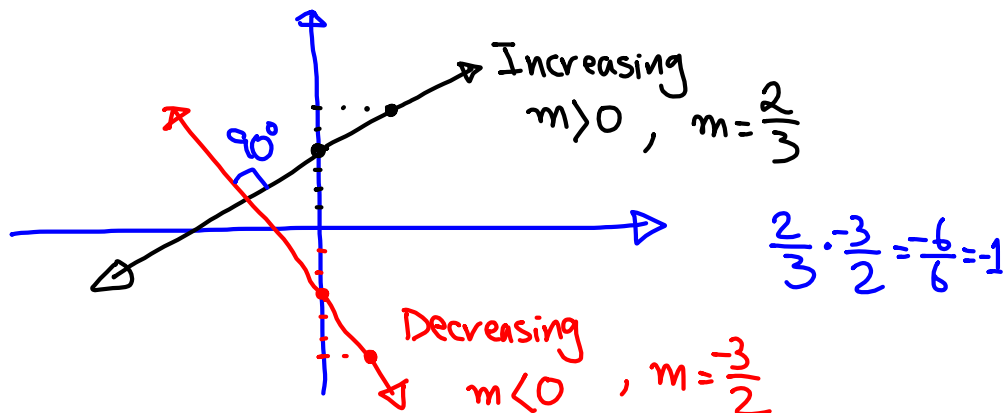$$y - 0 = \frac{3}{4}(x - 4)$$

$$y = \frac{3}{4}x - \frac{3}{4} \cdot 4$$

$$\boxed{y = \frac{3}{4}x - 3}$$

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find eqn of a line that contains $(0, -3)$
and is (perpendicular) to the line $y = \frac{2}{3}x + 4$.
Graph both lines. $y - -3 = \frac{-3}{2}(x - 0)$

$$y - y_1 = m(x - x_1) \qquad y + 3 = \frac{-3}{2}x \qquad \boxed{y = \frac{-3}{2}x - 3}$$

$90°$ Increasing
        $m > 0$, $m = \frac{2}{3}$

Decreasing
$m < 0$, $m = \frac{-3}{2}$

$$\frac{2}{3} \cdot \frac{-3}{2} = \frac{-6}{6} = -1$$

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find eqn of a line that contains $(-5, 7)$
with

① Zero slope
Horizonal line
$y = b$ $\boxed{y = 7}$

② No slope
Vertical line ←
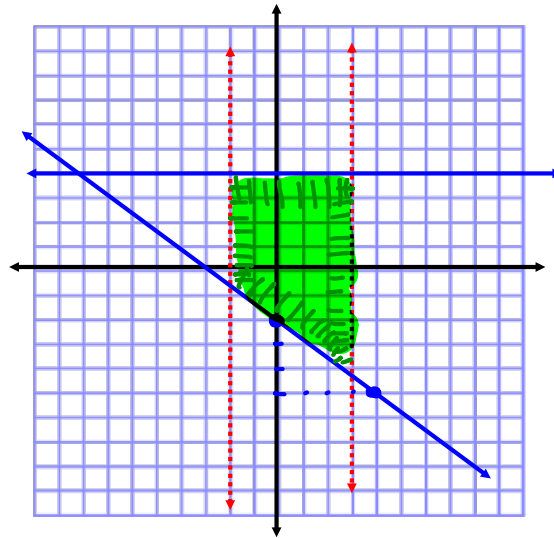$x = a$ $\boxed{x = -5}$

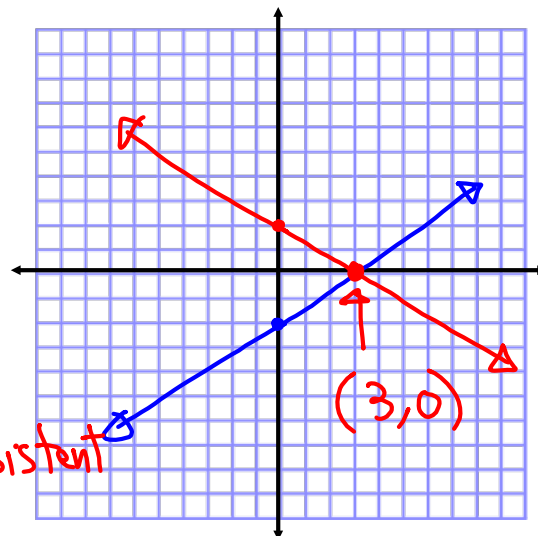③ undefined slope
$\boxed{x = -5}$

Graph & Shade

$\begin{cases} x < 3 \\ x > -2 \\ y \geq \frac{-3}{4}x - 2 \\ y \leq 4 \end{cases}$

Solve by graphing

$\begin{cases} y = \frac{2}{3}x - 2 \\ 2x + 3y = 6 \end{cases}$

when we have at least one solution, System is Consistent

(3,0)

when there is exactly one Solution, or when there is no Solution Equations are independent.
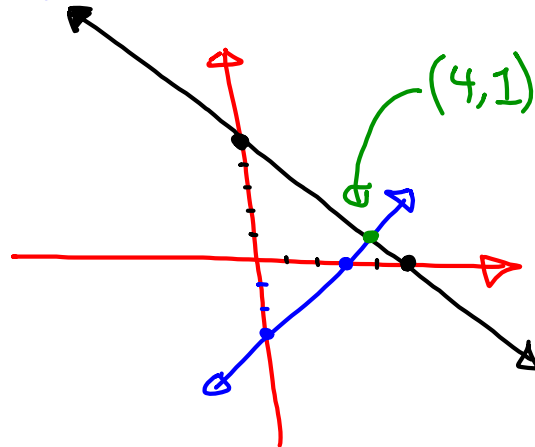
The Sum of <u>two numbers</u> is 5.
Their difference is 3. → x & y
Set-up a system of linear eqns in two
variables. Use graphing to solve

$$\begin{cases} x + y = 5 \\ x - y = 3 \end{cases}$$

System → Consistent

Eqns → independent

(4,1)

Solve by Subs.

$$\begin{cases} 3x + 2\boxed{y} = 4 \\ y = \boxed{3-x} \end{cases} \Rightarrow \begin{array}{l} 3x + 2(\boxed{3-x}) = 4 \\ 3x + 6 - 2x = 4 \\ \boxed{x = -2} \end{array}$$

at least one
Soln → System is
         Consistent
———————————
Exactly one Soln
  → Eqns are independent.

$y = 3 - (-2)$

$y = 3 + 2 \quad \boxed{y = 5}$

Final Ans $(-2, 5)$

Solve by Subs.

$$\begin{cases} 4x - 3\boxed{y} = 7 \\ \boxed{y} = \boxed{\dfrac{4}{3}x + 2} \end{cases}$$

No Soln

System is inconsistent.

Eqns are independent.

$$4x - 3\left(\frac{4}{3}x + 2\right) = 7$$

$$4x - 3\cdot\frac{4}{3}x - 3\cdot 2 = 7$$

$$4x - 4x - 6 = 7$$

$$-6 = 7$$

false

$$\varnothing$$

Solve by Subs.

$$\begin{cases} 3x + 5y = 10 \\ y = \dfrac{-3}{5}x + 2 \end{cases}$$

Infinitely Many Solns.

System is constistent

Eqns are dependent.

$$3x + 5\left(\frac{-3}{5}x + 2\right) = 10$$

$$3x + 5\cdot\frac{-3}{5}x + 5\cdot 2 = 10$$

$$3x - 3x + 10 = 10$$

$$10 = 10$$

True

Solve by Elimination Method:

$\begin{cases} 3x + \cancel{y} = 4 \\ 2x - \cancel{y} = 6 \end{cases}$

$\overline{5x \qquad = 10}$

$\boxed{x = 2}$

$3(2) + y = 4$

$6 + y = 4$

$\boxed{y = -2}$

$(2, -2)$

$\begin{cases} 3x + 2y = 6 \\ 2\,\big|\, x - y = 2 \end{cases} \Rightarrow$

$2 - y = 2 \Rightarrow \boxed{y = 0}$

$\begin{cases} 3x + 2y = 6 \\ 2x - 2y = 4 \end{cases} \rightarrow \boxed{x = 2}$

$\overline{5x \qquad = 10}$

$(2, 0)$

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the perimeter of a rectangular garden is 32 meters.

The length is (1 m longer than) twice its width. Use system of linear eqns to find its dimensions.

$\begin{cases} 2L + 2W = 32 \\ L = 2W + 1 \end{cases}$

W ▭ L
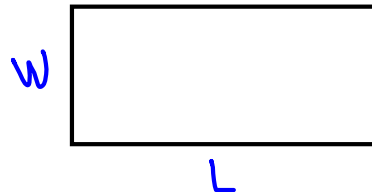
$2(2W + 1) + 2W = 32 \quad \rightarrow W = 5, \; L = 11$

$4W + 2 + 2W = 32$

$6W = 30$

5m by 11m.

Solve by Elimination

$2 \begin{cases} 2x + 3y = 5 \\ 3x - 2y = 14 \end{cases} \Rightarrow \begin{cases} 4x + 6y = 10 \\ 9x - 6y = 42 \end{cases}$

$\overline{13x = 52}$

$2(4) + 3y = 5$

$\boxed{x = 4}$

$3y = 5 - 8$

$\boxed{y = -1} (4, -1)$

The sum of two angles is $90°$.

Twice one plus 5 times the other is $360°$.

find both angles

Angles are Complementary.

$-2 \begin{cases} x + y = 90 \\ 2x + 5y = 360 \end{cases}$

$\begin{cases} \cancel{-2x} - 2y = -180 \\ \cancel{2x} + 5y = 360 \end{cases}$

$x + 60 = 90$

$\boxed{x = 30}$

$\overline{3y = 180}$

$\boxed{y = 60}$

$\boxed{30° \And 60°}$

The sum of two angles is 180°
"They are supplementary".

the difference of $\boxed{\text{4 times one}}$ and $\boxed{\text{3 times}}$ $\boxed{\text{the other one}}$ is 160°.

$3\begin{cases} x + y = 180 \\ 4x - 3y = 160 \end{cases}$

$\begin{cases} 3x + 3y = 540 \\ 4x - 3y = 160 \end{cases}$

$7x = 700$

$\boxed{x = 100}$

$100 + y = 180$

$\boxed{y = 80}$

$\boxed{100° \ \& \ 80°}$

Rafael has \$3.50 in quarters and nickels. he has 8 more quarters than nickels. How many of each?

$x \rightarrow$ # of quarters
$y \rightarrow$ # of nickels

$\div 5 \begin{cases} 25x + 5y = 350 \\ x = y + 8 \end{cases}$

$\begin{cases} 5\boxed{x} + y = 70 \\ x = y + 8 \end{cases}$

$5(y + 8) + y = 70$

$5y + 40 + y = 70$

$6y = 30 \quad \boxed{y = 5} \rightarrow \boxed{x = 13}$

$\boxed{13 \text{ Quarters} \ \& \ 5 \text{ Nickels}}$

$13(.25) + 5(.05) =$

A collection of 57 coins is made up of quarters & dimes has a value of $9. How many of each?

$x \rightarrow$ # quarters

$y \rightarrow$ # Dimes

$$\begin{cases} x + y = 57 \\ 25x + 10y = 900 \end{cases} \div 5$$

$$-2\begin{cases} x + y = 57 \\ 5x + 2y = 180 \end{cases}$$

$\boxed{x=22}$

$$\begin{cases} -2x - 2y = -114 \\ 5x + 2y = 180 \end{cases}$$

$$3x = 66$$

22 Quarters
&
35 Dimes

School sold some tickets for a play and collected $265.

$K \rightarrow$ # of kids, $A \rightarrow$ # of Adults

kid's ticket $\rightarrow$ $5

Adult's ticket $\rightarrow$ $10

$$\div 5\begin{cases} 5K + 10A = 265 \\ A = 2K - 16 \end{cases}$$

the number of adults was 16 fewer than twice the number of kids. How many of each?

$$\begin{cases} K + 2A = 53 \\ A = 2K - 16 \end{cases}$$

$K + 2(2K-16) = 53$

$K + 4K - 32 = 53$

$5K = 85$

$\boxed{K=17}$

$A = 2(17) - 16$

$= 34 - 16$

$\boxed{A=18}$

Irene found 40 bills, Some fifty-dollar bills, Some twenty-dollar bills.

Total value $1160. How many of each?

$x \to$ # of 50-dollar bills
$y \to$ # of 20-dollar bills

$\Rightarrow \begin{cases} x + y = 40 \\ 50x + 20y = 1160 \end{cases}$ ÷10

$-2 \begin{cases} x + y = 40 \\ 5x + 2y = 116 \end{cases}$

$\overline{}$

$3x = 36$

$\boxed{x = 12}$

$\boxed{y = 28}$

12   50-dollar bills
         &.
28   20-dollar bills

Aaron spent $270 in 10-dollar bills &. 5-dollar bills.

He used 6 less Fives than Tens.

How many of each?

$x \to$ # of 10-dollar bills
$y \to$ # of 5-dollar bills

$÷5 \begin{cases} 10x + 5y = 270 \\ y = x - 6 \end{cases}$

$\begin{cases} 2x + \boxed{y} = 54 \\ y = \boxed{x - 6} \end{cases}$

$2x + \boxed{x - 6} = 54$

$3x = 60$

$\boxed{x = 20}$ $\boxed{y = 14}$

20   $10 bills
14   $5 bills

SG 10 is due tomorrow.

SG 11 Due Thursday.

Exam II is Thursday.

Mixture Problems:

A lab assistant needs 70 liters of 8% alcohol soln. There are unlimited supply of 5% & 12% alcohol solns. How many liters of each?

5%	+	12%	=	8%
x		y		70

$$\begin{cases} x + y = 70 \\ \frac{5}{100}x + \frac{12}{100}y = \frac{8}{100} \cdot 70 \end{cases} \quad 100$$

$$-5\begin{cases} x + y = 70 \\ 5x + 12y = 560 \end{cases}$$

$$\begin{cases} -5x - 5y = -350 \\ 5x + 12y = 560 \end{cases}$$

$$\overline{\qquad 7y = 210}$$

$$\boxed{y = 30}$$

30 liters of 12% Soln
&
40 liters of 5% Soln.

How many cups of pure Sour Cream must be mixed with 12 Cups of 40% Sour Cream to get a 60% Sour Cream?

$$\boxed{100\%} \; + \; \boxed{40\%} = \boxed{60\%}$$

$$x \qquad\qquad 12 \qquad\qquad y$$

$$\begin{cases} x + 12 = y \\ \dfrac{\cancel{100}}{\cancel{100}}x + \dfrac{\cancel{40}^{2}}{\cancel{100}_{5}}\cdot 12 = \dfrac{\cancel{60}^{3}}{\cancel{100}_{5}}\cdot y \end{cases} \qquad \begin{cases} x + 12 = y \\ \;^5 x + \dfrac{2}{5}\cdot 12 = \dfrac{3}{5}y \end{cases}$$

$$\begin{cases} x + 12 = y \\ 5x + 24 = 3\boxed{y} \end{cases}$$

$$5x + 24 = 3(x + 12)$$

$$5x + 24 = 3x + 36$$

$$2x = 12 \qquad \boxed{x = 6}$$

we need 6 cups of pure Sour Cream to mix with 12 cups of 40% Sour Cream to get 18 cups of 60% Sour Cream.

We need 12 liters of 36% acid Soln.

We have two Solns, 45% acid & 18% acid.

How many liters of each?

$$\boxed{45\%} + \boxed{18\%} = \boxed{36\%}$$

x y 12

$$\begin{cases} x + y = 12 \\ 100\left(\dfrac{45}{100}x + \dfrac{18}{100}y\right) = \dfrac{36}{100} \cdot 12 \end{cases}$$

$$\begin{cases} x + y = 12 \\ \div 9 \quad 45x + 18y = 36 \cdot 12 \end{cases} \Rightarrow \begin{cases} x + y = 12 \\ 5x + 2y = 48 \end{cases} \quad \begin{array}{l} x = 8 \\ y = 4 \end{array}$$

8 liters of 45% Soln.

4 " " 18% ".